Increasing Injection Molding Speed through Cooling System Geometry Optimization

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One of the aspects determining the speed rate, and therefore the economic viability, of an injection molding process is the efficiency of the heat transfer between the injected part and the mold. The quicker the temperature of the part drops below the threshold value at which it can be ejected from the mold, the sooner the mold can be closed and the injection of next part can begin. In this article is presented the solution to the highly constrained structural design optimization problem of maximizing the heat exchange surface area of the cooling system of an injection mold, an exercise aimed at increasing the speed of an existing molding system. The author described how a systematic design search via an evolutionary heuristic specifically designed for such highly constrained design problems can yield substantial increases in cooling area, while allowing the relatively straightforward implementation of the constraints that ensure the functionality of the system.

Keywords: Evolutionary optimization, Injection molding process, Punctuated equilibrium, Cooling system

We highlight here the work of Tang et al. [3] aimed at minimizing the average temperature of an injected part. In [4] is described a computer-aided optimal design system for improving the performance of a cooling system for injection molding. In [5] is presented an optimal design of heating channels for a rapid heating cycle injection mold; the distances between the neighboring heating channels were the main design variables. Recently, the plastic injection mold with complex cooling channels can be produced by Rapid Prototyping [6]. From this category of researches we outline the following studies. Sachs et al. [7] applied a Solid Freeform Fabrication Process called Stereolithography (or Three Dimensional Printing – 3DP) to increase both the quality and the rate of injection molding through improved control of the temperature of the tool during the molding process. Xu et al. [8] presents a systematic method for the design of conformal cooling channels for tooling. A new algorithm for recognition of the specific features corresponding to a cooling system design is developed in [9]. The features-based algorithm decomposes the analyzed part into simpler shape and after that the cooling sub-circuits are generated to provide the required cooling function for each recognized feature. An extension of this work is described in [10]. Li et al. in [11] present a configuration space (C-space) method, developed to support the automation of the layout design of cooling channels.

In this paper is considered a specific design problem, where the optimization of the geometry of the cooling system must be performed in a highly constrained design space. This feature of the problem has two notable consequences. First, a broader viewpoint is needed, even encompassing an analysis of the dynamics of the ejection process, which, as we shall see, is tightly coupled with the design of the cooling network. Second, the complex topology of the feasible subspace(s) of the design space demands an optimization heuristic specifically designed for effective constraint-handling.

We therefore employ a two-phase evolutionary algorithm i.e. 2PhEA – Tudoș et al. [12, 13] based on the

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paradigm of ‘punctuated equilibrium’ [14,15], where periods of stasis periodically interrupt the process of evolution under the selective pressure of the objective function and subject the population to evolution towards constraint satisfaction.

In the following section we shall describe this heuristic (i.e. the 2PhEA) in more detail, followed, by a detailed discussion (Section 3) regarding the statement of the design problem. In section Section 4 are presented the results of its application. We conclude the discussion with reflections on possible extensions to the study, as well as possible implications in other areas of engineering design.

A Two-Phase Evolutionary Algorithm (2PhEA)

There is evidence in a variety of fields that Evolutionary Algorithm (EA) searches can be effective in this type of structural design application with a very large number of constraints. Additionally, the requirement for handling a mixed variable set, as well as for keeping a global outlook across a potentially multi-modal design space, makes EAs almost uniquely suited for this application. We have therefore selected this class of heuristics for solving the design optimization problem (i.e. the optimal design of the cooling system) described in this paper. Our 2PhEA [12,13] inspired from the evolutionary concept of “punctuated equilibrium” work as follows:

(a) **Phase one**: Evolution towards feasibility. Simulate one generation of artificial evolution driven exclusively by the selective pressure of the constraints;

(b) **Feasibility test**: Does the ratio of feasible individuals exceed a pre-set threshold value? If not, return to (a); if yes, return to (c).

(c) **Phase two**: Evolution towards high performance and feasibility. Simulate one generation of artificial evolution (following the canonical template of Goldberg [16]), driven by the objective function penalised by the constraints (this step is equivalent to the standard constrained evolutionary search, based on the fundamental principles introduced by Fiacco and McCormick [17]—our implementation follows the guidelines of Keane and Nair [18]).

(d) If an objective-related convergence criterion is met, stop. Otherwise, if the percentage of feasible individuals has dropped below a certain threshold, return to (a); if not, return to (c).

In the following we present, in short, how to determine the treatment of the cooling system in isolation from the rest intricate network of constraints, which precludes the treatment of the cooling system in isolation from the rest.

The **Design Problem**

We consider the problem of the optimal design of the cooling system of a mold for a simple truncated cone shaped flower pot (fig. 1, a).

![Figure 1](image_url)

**Fig. 1** (a) Simple truncated cone shaped flower pot; and (b) core side of the mold, including the five cooling circuits and the ejector (semi-transparent frontal and isometric views)

The mold core side illustrated in figure 1, b represents the design currently in use – we seek to increase its performance by maximizing the overall cooling area of the channels, without impairing the functioning of the ejection system. The complexity of this problem lies in the intricate network of constraints, which precludes the treatment of the cooling system in isolation from the rest.

In order to use these constraints in our algorithm we needed to aggregate them in the following form:

$$\begin{align*}
G(x) &= \begin{cases}
0, & g_i(x) \leq 0 \\
g_i(x), & g_i(x) > 0 \\
0, & g_i(x) < 0 \\
0, & g_i(x) = 0 \\
g_i(x), & g_i(x) \neq 0
\end{cases} \\
i &= 1, n_s \\
nen_i + 1, n_s + n_i \\
nen_i + n_i + n_s \\
en_i + n_s + 1, n_s + n_i + n_s
\end{align*}$$

where: $\varepsilon$ represents a very small positive quantity.

In each phase, for each individual a so called score is computed. The partial score of an individual (from those $N$ individuals of the population) $x_j, (j = 1, N)$, regarding to the constraint $i$, $(i = 1, n_s + n_i + n_s)$ is computed as follows:

$$S(x_j) = \sum_{i=1}^{n_s + n_i + n_s} P_{Si}(x_j)$$

Eventually, the (individual) score of each individual $x_j, (j = 1, N)$ of the population is:

$$S(x_j) = \sum_{i=1}^{n_s + n_i + n_s} P_{Si}(x_j)$$

Obviously, any feasible individual has null score. During **Phase 1** the population is sorted by the score, and during **Phase 2**, the population is sorted by score and objective value.

Regarding the implementation of the algorithm, we opted for Cambrian [12,13], a tool incorporating numerous recent developments in evolutionary search technology, whose name hints at the inclusion of the paradigm of “punctuated equilibra” in the design of its underlying heuristic.
of the mold. In particular, the pneumatic ejector (fig. 6), which strips the molded part off the core, shares the internal volume of the core with the cooling system (fig. 1, b) and therefore its dimensions have a strong influence on the layout and sizing of the cooling channels. In turn, the sizing of the ejector determines the ejection velocity, which therefore links it to the overall dimensions of the open mold, as the trajectory of the ejected part must be such that it does not strike the cavity side of the mold (fig. 2). It is the analysis underpinning this constraint that we discuss next.

The Flight of the Molded Part

A critically important boundary of the design space of the ejector (fig. 6) and, implicitly, the cooling system, is determined by the constraint that the part must not be struck so hard (in an attempt to expedite its evacuation) that it strikes the cavity side of the mold in its flight. The computation of this constraint is a rather complex problem, due to the tumbling nature of the flower pot flight (flower pots are not designed for a smooth and steady glide!). An added challenge is the need for computationally inexpensive analysis (dictated by the need to run extensive optimization heuristics on the problem, requiring thousands of re-runs of the model), which precludes detailed numerical analysis of the flow around the pot (say, solving the Euler flow equations on a moving mesh that travels with the part, combined with the computation of its equations of motion). We have therefore utilized the following simplified method of calculation.

Consider the sketch in figure 3, which depicts the molded, ejected part, frozen in time in a moment of its flight. Upon it acts the force of gravity $m_p \cdot g$ and the aerodynamic drag $F_{rez}$ in the points C and P respectively – the tumbling aspect of the motion results from the moment generated by these forces. The angles between the horizontal, the part axis of symmetry S, and the tangent to the flight trajectory T are denoted by $\phi$ and $\psi$ respectively. The axis of symmetry pierces the bottom of the pot in D. It can be shown that the moment of inertia of the flower pot with respect to an axis going through the center of gravity is, applying the Huygens-Steiner theorem to a pot broken down into its base (subscript '1') and side (subscript '2'):

$$J_C = J_{1AD} + J_{2AD} - m_p \cdot DC^2$$

where:

$$J_{1AD} = \frac{1}{4}\pi \cdot \rho_m \cdot r^4 \cdot e$$

$$J_{2AD} = \frac{\rho_m \cdot \pi \cdot h}{R - r} \int_0^\rho \left[ \frac{1}{4} \left( x^2 - (x - y)^2 \right) + \frac{h^2}{(R - r)^2} \cdot (x - r)^2 \cdot (x - (x - r))^2 \right] dx$$

$R$ and $r$ denote the base and apex diameters of the cone respectively; $h$ is the cone height; $e$ is its wall thickness; and $\rho_m$ represents the density of the plastic material of the flower pot.

In order to estimate the drag we must also calculate the cross-sectional area of the body, or, to be more specific, the area of its projection onto a plane perpendicular to the velocity vector. Additionally, we need to know the location of the center of pressure (the distance between this and the center of gravity is, after all, a measure of the rotational tendency of the part in flight) – we shall denote this by $X_p$ in a system of coordinates attached to the part. Once again, due to the rotational component of the motion of the part we seek the area and $X_p$ as functions of $\psi$ (fig. 4 for an illustration). With $A(\psi)$ computed, we can calculate the drag $F_{rez}$ (also a function of $\psi$) as:

$$F_{rez} = c_d \cdot A(\psi) \cdot \rho_{air} \cdot \frac{v^2}{2} = c_d \cdot A(\psi) \cdot \rho_{air} \cdot \frac{x^2 + y^2}{2}$$

where $c_d$ is the drag coefficient and $\rho_{air}$ is the density of air.

With all these building blocks in place, we can write the equations of the flight of the part in the $xy$ plane, namely Euler’s equations of the motion of a rigid body:

$$\begin{align*}
  m_p \cdot \ddot{x} &= -F_{rez} \cdot \cos(\phi + \psi) \\
  m_p \cdot \ddot{y} &= -m_p \cdot g + F_{rez} \cdot \sin(\phi + \psi) \\
  J_C \cdot \ddot{\phi} &= F_{rez} \cdot [d_{\phi}(\phi) - d_{\phi}(\phi)]
\end{align*}$$

where $d_{\phi}(\phi)$ and $d_{\phi}(\phi)$ denote the levers of the drag force and gravity and $J_C$ is the moment of inertia of the pot. The following boundary conditions are applied at time $t = 0$:

$$x = y = 0; \quad \psi = 0; \quad \dot{x} = v_0; \quad \dot{y} = 0; \quad \dot{\psi} = 0$$

Through appropriate intermediate variables the system (9) can be converted into a set of six first order differential equations.
equations, which can be solved readily through a Runge-Kutta scheme.

Figure 5 shows some sample results of simulations based on this equation, with different ‘launch speeds’. With this low computational cost simulation capability in place, for each candidate design we can compute the acceptable range for the speed at which the part must be struck. We have two separate boundaries constraining the design space here. On the one hand, at the lower end of the speed range we must make sure that the impact is large enough to dislodge the part from the mold. At the same time we must check that it will not strike the other half of the open mold in its flight.

Tracing the chain of interlinked constraints that ultimately impact the cooling area, with the ejection speed constraint in hand, we now step down a level, to the modelling of the actual ejection process.

**Ejection Dynamics**

Figure 6 is an illustration of the part stripping mechanism. This is, essentially, a simple pneumatic ejector, comprising two air ducts (one for the injection of the air during the ejection stroke and the other for the injection of the return stroke air) and a piston moving in an axial direction in a cylinder drilled into the core side of the mold.

Fig. 4 Cross-sectional area projection as a function of \( \psi \). \( \psi_0 \) denotes the angle between the axis of symmetry and the velocity vector where the side of the flower pot becomes parallel with the velocity vector – this is the point past which the airflow begins to ‘see’ both the apex and the base of truncated cone.

![Cross-sectional area projection](image)

where \( m_p \) is the mass of the part, \( m_a \) is the combined mass of the moving parts of the ejector and \( c_{damp} \) is a damping coefficient accounting for the energy lost during the impact as a result of the deformation of the part (we have established this experimentally as \( c_{damp} = 0.8 \)). The only unknown here is \( v_{ai} \), which we determine by applying Newton’s second law of motion to the piston:

\[
(12) \quad m_e \cdot \ddot{x}_e = p_1 \cdot A_1 - p_2 \cdot A_e - F_f \cdot A_e - F_F.
\]

Here the indices ‘1’ and ‘2’ refer to the expansion chamber and the passive chamber of the cylinder, the \( p \) denoting pressure, the \( A_e \) denoting piston surface area (\( A_e \) is \( A_1 \) minus the cross sectional area of the ejector rod) and the \( F_F \) represents the friction forces on the piston and the rod respectively.

In order to compute the pressures, we need to combine (12) with the differential equations of the gas dynamics of the ejection process, obtaining the system:
where \( k \) is the adiabatic index, \( \mathcal{R} \) is the gas constant, \( T \) is the temperature of the air, \( p_r \) is the pressure of the pneumatic circuit, \( x_a \) is the stroke of the piston, \( A_i \) and \( A_o \) are the air inflow and outflow cross-sectional areas respectively of the two air ducts feeding the ejector, \( V_{01} \) and \( V_{02} \) are the volumes of the two chambers (either side of the piston) before the start of the ejection stroke.

Once more we apply the Runge-Kutta scheme for the solution of the system, setting the initial conditions (at \( t = 0 \)) as:

\[
x_a(0) = 0; \quad v_a(0) = 0; \quad p_i(0) = p_r; \quad p_o(0) = p_a
\]  

where \( p_a \) is the ambient pressure.

Constraints

The design exercise discussed here is a good example of how a deceptively simple geometry can yield a highly complex structural design optimization problem by virtue of the complex functional and geometrical relationships that must be satisfied by the solution. We have discussed some of these constraints in detail in the previous subsections – here is the complete list:

C1. The diameter of the ejector rod should be small enough to ensure that there is enough working volume in the cylinder around it for the return stroke.

C2. An assembly constraint: there must be sufficient difference between the diameters \( d_{d1} \) and \( d_{d2} \) to ensure the damage-free mounting of ring 4 (refer to fig. 6 for all three items, as well as those referred to under the following constraints).

C3. A simple geometrical constraint:

\[
g_a(x) = D < d_c
\]  

C4. A functional constraint:

\[
g_f(x) = d_r - D < 2 \cdot h
\]  

C5. A constraint on the wall thickness of the cylinder:

\[
g_w(x) = \begin{cases} 
  d_c - D & \text{if } d_c \leq 25 \\
  d_r - D & \text{if } d_r \geq 3 
\end{cases}
\]  

C6, C7. Geometrical constraints:

\[
g_c(x) = \frac{d}{d_p} \leq 0.2
\]  

C8. The impact pressure must be less than the crushing strength of the pot material to ensure that the part is not damaged by the ejector in the mold stripping process.

C9. The post-impact speed of the pot (as resulting from eq. 13) is sufficiently large to dislodge it from the mold.

C10. The post-impact speed of the pot is small enough to avoid a collision with the open half of the mold (see Section 2.1 for the relevant analysis).

C11, C12. The diameter of the circle enveloping all cooling channels must be less than a specified maximum (this relates to the minimum wall thickness of the mold itself) and greater than a specified minimum – refer to figure 1, b.

These 12 constraints combine to create a design space with complex, often non-linear boundaries, which severely limit the extent of the feasible subspace. This demands a specialised optimization heuristic – we discuss in detail our choice of optimizer in section 2. First though, to conclude the discussion of the design problem itself, we present the parameters, which define the dimensions of the design space.

Parameterization

The extent of a design space and therefore the cost of its exploration is an exponential function of the number of its dimensions. This 'curse of dimensionality' demands a great deal of care in the selection of the design variables for a particular optimization problem. The resulting parameter list and parameter value ranges are a statement of the designer's bias in a trade-off between flexibility (that is, the range of possible designs to be considered) and conciseness. Here is the list of parameters (table 1) we have selected for the cooling system design problem (with reference to fig. 6), along with the number of possible, discrete values they may take.

The objective

The ultimate goal of the design of the cooling network is the maximization of the speed of the heat exchange process (with the purpose of expediting the injection molding process by reducing the time between subsequent parts). Here we use the overall heat exchange area as a surrogate for the cooling time, as this is likely to have a very similar variation (with respect to the design variables), while its calculation being much faster, enabling a more extensive optimization process.

Results and conclusions

The baseline design (shown in fig. 2) had an overall cooling surface area of 1.549 \( \times 10^{-5} \text{m}^2 \) as a result of running the evolutionary search described above we successfully managed to increase the area to 2.026 \( \times 10^{-5} \text{m}^2 \), a change of 30.8%, with the number of cooling channels increasing from five to ten. It is worth noting here that while adding

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Range</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_c )</td>
<td>[0, ..., 63]</td>
<td>External diameter of the cylinder. Integer standardized values.</td>
</tr>
<tr>
<td>( d )</td>
<td>[0, ..., 63]</td>
<td>Diameter of ring 1. Integer standardized values.</td>
</tr>
<tr>
<td>( D )</td>
<td>[0, ..., 63]</td>
<td>Diameter of ring 2. Integer standardized values.</td>
</tr>
<tr>
<td>( d_{d1} )</td>
<td>[0, ..., 63]</td>
<td>Diameter of ring 5. Integer standardized values.</td>
</tr>
<tr>
<td>( n_c )</td>
<td>[5, 7]</td>
<td>The number of the cooling channels.</td>
</tr>
</tbody>
</table>

Table 1: The 5 Design Variables Describing the Cooling System

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cooling channels may seem like a fairly intuitive way of enhancing the heat exchange process, when examined in the light of the 12 constraints of the problem, the question is a much more difficult one. On the one hand, space constraints mean that adding more channels reduces the ‘per channel’ heat exchange area. Additionally, the complex network of constraints makes finding a feasible solution (let alone a globally optimal feasible solution) quite challenging.

The proliferation of complicated constraints is almost inevitable in problems considered at an industrially relevant level of detail. This may curb basic optimization attempts, as the generation of feasible designs might be an issue in itself, even before the optimization of the actual objective is taken into consideration. We have reported on this study here as an illustration of the fact that specialised optimization tools capable of ‘unweaving’ complex networks of constraints even over large design spaces can bring very significant performance benefits on such design problems. The roadmap presented here may provide a useful template for the solution of similar problems in different fields of mechanical design.

References