ABS 3D Printed Facial Study Model Using Hermite Matrix Interpolation for Manufacturing Facial Epistasis

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Facial epistasis represent facial prosthetic components made of acrylic or silicone plastics, which have no other function than to rebuild the lost region of the face in order to partially restore the physiognomy. A mandatory step in the manufacturing process of epistasis is represented by fingerprinting the tissue defect and the entire facial region and making a study model as basis for designing the epistasis.

Keywords: facial epistasis, prosthetic components, acrylic or silicone plastics

Facial physiognomy can be altered by the presence of facial post-surgical defects. Facial epistasis (fig 1.) represents facial prosthetic components made of acrylic or silicone plastics, which have no other function than to rebuild the lost region of the face in order to partially restore the physiognomy. The manufacturing process of a facial epistasis requires an interdisciplinary collaboration between the clinicians involved in the treatment of the deformity and technician specialized in manufacturing epistasis. That's why a mandatory step is represented by fingerprinting both the tissue defect and the entire region and making a study model as basis for designing the epistasis.

Classic fingerprinting using fingerprinting materials has the drawback that modifies the facial soft tissues, recording a situation different from the real one. Modern technologies allow optical fingerprinting either by using a 3D scanner or by computed tomography followed by 3D reconstruction. The latter records simultaneously the spatial distribution both of soft tissues and of hard ones which are the support of soft tissues support and of the epistasis itself [1, 2].

Three-dimensional virtual reconstruction is based on sections of computed tomography. These sections represents sections of the patient's anatomy located at a certain distance imposed by the maximum dose of X-ray radiation which the patient can bear [3, 4, 11, 12]. Therefore the missing information between two consecutive section images has always to be obtained through mathematical interpolation methods.

The idea of three-dimensional reconstruction led to the application of interpolation methods in software algorithms: search for a smooth and continuous function, which in some points (control or standard) takes given values. An essential prerequisite, which is desirable to be true in any context, is that interpolation function to go through all the points set out and in addition to satisfy the conditions of continuity and class C derivability in these points, in order to achieve a certain level of smoothness and prevent undesired corners or ditches.

The imaging theory backbone is still integral geometry. In the three-dimensional (3D) case, we shall use the approximate and exact formula of inverting for interpolation methods using Hermite-type polynomials.

The main objective of the research is to improve the mathematical algorithms used in facial reconstruction programs, such as specialized CAD/CAM software, which implement these algorithms, to provide 3D reconstructed model, later printed in ABS (acrylonitrile butadiene styrene), which approximates the human model better than the standard methods previously used.

The paper presents a mathematical interpolation method used in three-dimensional reconstruction of sections of computerized tomography of the virtual model of the patient’s face such that it can be printed of ABS (acrylonitrile butadiene styrene) to represent a physical study model for the construction of the facial epistasis [5, 6, 9, 10, 12].

Experimental part

CT slices of a patient with left eyeball resection where used for 3D reconstruction of the virtual model of the left orbital region using own reconstruction software based on own mathematical optimized algorithms. 3D virtual model obtained was 3D printed using ABS. (fig 2.). Acrylonitrile butadiene styrene is a thermoplastic material with the melting termic interval situated around a low temperature, suitable for fused deposition modelling, the most common technology for 3D printing. 3D printing in opposition to classic fingerprinting preserves all dimensions, making the study model an exact copy of the facial region, eliminating unwanted contractions and dilations due to bulk material use.

The appropriate mathematical modelling algorithms implemented in 3D reconstruction software uses, generally, Bezier or B-spline curves approximation. These curves do not pass through the specified control points, except the first and last point. They are only influenced by these points,
therefore are not really “interpolation” curves. Even more, Bezier curves are included in the polygon defined by the control points. Other points are used to control the shape of the curve. Unlike spline curves, Bezier curves take advantage of an easier writing of the conditions of smoothness. The slope and the tangent to the first and last point depend on the second and final control point [7, 8].

Hermite interpolation method approach in 3D reconstruction is accomplished by knowing values and derivatives of the function at the selected points. So it takes 4 equations to determine the four unknowns:

\[
x(u) = a u^3 + b u^2 + c u + d \\
x'(u) = 3a u^2 + 2b u + c
\]

To create a composite curve, one can use the node at the end of a curve so that the start of the next curve and the tangent vector to the curve is common. Considering x as a variable, is given the polynomial of degree 3 and its derivative

\[
x(u) = a u^3 + b u^2 + c u + d \\
x'(u) = 3a u^2 + 2b u + c
\]

Given \( x, x_i, x_{i+1}, x', x'_{i+1} \), the system will be solved for \( a, b, c, d \):

\[
\begin{align*}
x(0) &= x_i = d_i \\
x(1) &= x_{i+1} = a_i + b_i + c_i + d_i \\
x'(0) &= x'_i = c_i \\
x'(1) &= x'_{i+1} = 3a_i + 2b_i + c_i
\end{align*}
\]

with the solution:

\[
\begin{align*}
a_i &= 2(x_i - x_{i+1}) + x'_{i+1} \\
b_i &= 3(x_{i+1} - x_i) - 2 x'_i - x'_i - 1 \\
c_i &= x'_i \\
d_i &= x_i
\end{align*}
\]

The coefficients can be expressed as linear combinations of geometric information:

\[
\begin{bmatrix}
a_x \\ b_x \\ c_x \\ d_x
\end{bmatrix} = \begin{bmatrix}
2 & -2 & 1 & 1 \\
-3 & 3 & -2 & -1 \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
x_i \\ x_{i+1} \\ x'_i \\ x'_{i+1}
\end{bmatrix}
\]

The x component of the Hermite curve will be represented as:

\[
x(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix}
a_x & b_x & c_x & d_x
\end{bmatrix}
\]

\[
= \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix}
2 & -2 & 1 & 1 \\
-3 & 3 & -2 & -1 \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
x_i \\ x_{i+1} \\ x'_i \\ x'_{i+1}
\end{bmatrix}
\]

By including now the components y and z:

\[
[x(u) y(u) z(u)] = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix}
a_x & a_y & a_z \\
b_x & b_y & b_z \\
c_x & c_y & c_z \\
d_x & d_y & d_z
\end{bmatrix}
\]

Denoting \( P(u) = [x(u) y(u) z(u)], u_i = [u^3 \ u^2 \ u \ 1], \)

\[
M = \begin{bmatrix}
2 & -2 & 1 & 1 \\
-3 & 3 & -2 & -1 \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0
\end{bmatrix} \quad \text{and} \quad P = \begin{bmatrix}
x_i \\ y_i \\ z_i \\ x_{i+1} \\ y_{i+1} \\ z_{i+1} \\ x'_i \\ y'_i \\ z'_i \\ x'_{i+1} \\ y'_{i+1} \\ z'_{i+1}
\end{bmatrix}
\]

we shall obtain \( P(u) = u^3 M \) where \( u^3 \) is the parameter, M is the matrix of coefficients and \( p_0 \) is the variable where geometric information is recorded.

Hence, each Hermite interpolation cubic spline is a linear combination of four mixed functions with the property \( P(u) = u^3 M \), where:

\[
P(u) = \begin{bmatrix}
2u^3 - 3u^2 + 1 \\
-2u^3 + 3u^2 - u \\
u^3 - 2u^2 + u \\
u^3 - u^2
\end{bmatrix} \begin{bmatrix}
p_0 \\ p_1 \\ p_2 \\ p_3
\end{bmatrix}
\]

Using the 3D printed model based on the algorithms described above, an ocular epistasis was manufactured through standard technique (fig 3.)

**Results and discussions**

By using interpolation techniques with blending functions of Hermite-type we can get 3D reconstruction algorithms that approximate the original object more precisely, due primarily to the fact that the interpolant walks through all desired points initially, the resulting curves being continuous (smooth) and derivable, so of the class \( C^1 \). However, if originally selected points are sufficiently distant from each other, the method will not work very well because the curve that will unite the two consecutive points might go away from the one coming from the real object, being in fact the graph of a polynomial function.

**Conclusions**

The mathematical interpolation method can be successfully used for the three-dimensional reconstruction of a virtual model of the patient’s face. By 3D printing a physical study model out of ABS (acrylonitrile butadiene styrene), the latter can be used during the manufacturing process of a facial epistasis. An accurate copy of the patients face has been obtained through interpolation of the missing data between consecutive computer tomography slices.

Study models obtained through this method can be successfully used for analyzing different cases and especially for interdisciplinary collaboration between the doctor and the technician but they can also be successfully used even during the modelling process of facial epistasis.

In future research, the authors propose to extend this method to a composed interpolation method which combines the accuracy of Hermite interpolation with harmonious lines of B-splines or Bezier approximation in order to obtain a 3D printed ABS plastic model, showing
smaller differences than that obtained by the method described here.

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