For all types of mechanical structures, no matter the functional, designing characteristics and manufacturing technologies, it is well known – the exceptions are very rare – the failures and fractures have a pronounced local character. The fracture is initiated in a point or in a very small region (in comparison with the overall dimension of the whole structure) and will extend and propagate till the whole structure becomes unsafe and unusable.

The whole structure will be designed according to some standards and specific procedures elaborated and accepted internationally, assuring a good availability and operational reliability of this structure in its entirety.

In order to increase the performances of the structure, its safety and life time, more accurate determinations are often necessary for the displacement and stress states configurations, for a great variety of local problems of structures.

These demands are more emphatic and become mostly imperative, especially for laminated composite structures. The failure and fracture mechanisms of composite materials and especially of laminated structures are different and more complex than usual homogeneous and isotropic materials.

Due to this fact, the modelling and numerical analysis of laminated composite structures must be done having as main purpose the determination of local stress states in the areas where stress variations with high gradients occur (i.e. supports, geometrical discontinuities, junctions etc.).

Based on the engineering practice, the authors conclude that the main and frequent local problems are those linked to discontinuities and junctions.

In this paper are presented general methodologies and shown as examples some solutions for both categories.

The modelling and the analysis of such problems could not be done using classical methods of calculus. The only possibility is offered by the Finite Element Method, which has to be adapted to the laminated composite specific.

Laminated composite structures with discontinuities

For the modelling and usual analysis of mechanical structures it is considered that the whole structure and its components accomplish one of the fundamental conditions of strength of materials – the condition of continuity. The great variety of problems in engineering practice yields sometimes situations where discontinuities appear in some components.

Discontinuities structures are local macro failures of the material continuum, having different dimensions and configurations. They have to be modeled and analyzed with other means than the classic ones by mechanical engineering, due to the complexity loads and the local stress state.

There are different mechanisms and conditions of appearance of discontinuities, such as:

- in the design process, when discontinuous welding straps are used or when different components of an assemblage have higher tolerance that could yield to excessive gaps;
- in the fabrication processes when cutting conditions are heavy, thermal treatments are improper or parts subjected to cutting processes are improperly constrained;
- in the assembly process, due to execution errors as: insufficient fastening of the assembly elements, improper gaskets, weldings or contacts between the elements of the structure;
- defects of the material, as: macro cracks, inclusions, shrinkage holes;
- in service, due to corrosion, wear or accidental overloads.

Modeling and analysis of discontinuities are different of the fracture mechanics approach due to the macro scale used in the approach, different from the micro scale approach in fracture mechanics.

The systematic and complete approach of the effects of discontinuities in mechanical structures is complex, very up-to-date and important for the assessment of the safety and integrity in the exploitation process of machines, devices, equipment, etc.

Fundamental for a discontinuity is its local character, its certain shape. The modelling and analysis of discontinuities could be done using methodologies and adequate techniques for local problems such as substructuring or submodelling and the use of finite element method.

The purpose of this paper is to underline the importance of the study of discontinuities in mechanical structures and to present a methodology of approach (one of the many that can be used). The case of plates made from laminated

Keywords: laminated composites, local stress states, macro failures, finite element method
composites was selected from the large field of discontinuities met in engineering practice.

Two types of plate were considered: a plane plate and a three dimensional plate of a certain shape. A discontinuity having the shape of a quadrilateral was defined in the resin layer of the material, that is a laminated steel-epoxy-aluminum composite. Several variants of loading and constraint conditions were considered, in order to get a bigger volume of information that may define a broader image of the stress and displacement states.

The authors hope that the conclusions of this work may justify the interest for modeling and analysis of discontinuities from mechanical structures.

Concepts for discontinuities. Examples

In order to clarify the general concept of discontinuity, the longitudinal section of a straight bar with circular cross section has been taken into account. An oblique hole of conical shape was drilled for fastening purposes.

No matter what the material is, loading and constraints, the stress and strain state cannot be obtained using the classic methods of mechanical engineering or strength of materials. The elaboration of a three dimensional finite element model is necessary. The choice of the element type and the mesh parameters depend on the purpose of the analysis. For example, one may want to obtain the maximum values of displacements, stresses and contact pressure between the bar and the bolt that is assembled into the conical hole.

Since the model is 3D, the modeling process, analysis and data processing, interpretation and presentation are laborious enough, although the problem may look very simple at the first sight.

Consequently, one can underline the great importance of the problems of discontinuities in mechanical engineering, and also the efforts and competence necessary to overstep the difficulties that appear in their modeling and analysis.

Features of the approach and resolution of discontinuities in laminated composites

In order to detail some aspects of modeling and analysis of discontinuities, the case of a plate made from a steel-epoxy-aluminum laminated composite was considered. A discontinuity (a gas bubble of a certain shape) was presumed to appear in the resin layer due to an imperfection of the manufacturing process of the material. X-rays or γ-rays may be used to detect the discontinuity and to find its shape and dimensions.

The effects of such a discontinuity may be determined by numerical calculations, using a specific methodology of finite element modeling and analysis. This methodology should consider the following very important aspects:

a. since the discontinuities have a local character, a sub model or a substructure of the model must be considered, ensuring thus the efficiency of the process [1];

b. the geometry of the model (key points, lines, surfaces, volumes) must be defined in such a way as to put in evidence the layers of the composite material, made of different materials; thus, the results of the finite element analysis will yield information concerning the stresses and strains in each layer and at the interfaces between layers;

c. the mesh must be done by creating the nodes and elements along each layer, as distinctive groups of elements with different physical, elastic and mechanical properties;

d. the model must be three dimensional and use brick-type elements [2, 3].

The other aspects of modeling and analysis are the common ones.

In order to illustrate and clarify the above mentioned considerations and principles, two examples of plates with discontinuities are presented: a plane plate and a 3D twisted plate. A discontinuity having a prismatic shape was considered. The measurement units are N and mm.

The materials from which the composite is made are considered as isotropic, homogeneous and with linear elastic behaviour, having the following elastic constants:

- for resin: $E = 2.44 \cdot 10^3 \text{ N/mm}^2$; $G = 1.2 \cdot 10^3 \text{ N/mm}^2$; $\nu = 0.46$;

- for aluminum: $E = 7 \cdot 10^4 \text{ N/mm}^2$; $G = 2.5 \cdot 10^4 \text{ N/mm}^2$; $\nu = 0.35$;

- for steel: $E = 2.1 \cdot 10^5 \text{ N/mm}^2$; $G = 8.1 \cdot 10^4 \text{ N/mm}^2$; $\nu = 0.3$.

Both plates were considered fixed on the surface lying in the OYZ plane. The loading was defined as two loads applied in the nodes of the surface opposite to the fixed one, and having the following values: a horizontal force $F_x = 21 \text{ kN}$ and a vertical force applied downwards $F_y = -4.2 \text{ kN}$.

Both models used tetrahedral 3D elements, having three degrees of freedom per node. A great volume of quantitative and qualitative information was obtained following the finite element analyses. Some of them being of greater interest are presented in the further paragraphs in suggestive, simple and accessible forms.

Example of a plane plate with a discontinuity in the resin layer

A rectangular plane plate with constant thickness was considered first. The general aspect is presented in figure 1.a and the discontinuity in the resin layer (fig. 1b).

The model of the plate had 864 nodes and 3100 elements. The stress fields in the plate are presented in the next figures as follows:
- in figure 3 - von Mises equivalent stress field in a plane parallel with OXZ, having Y = 15 mm;
- in figure 4 - $\sigma_x$ stress field in a plane parallel with OXZ, having Y = 15 mm;
- in figure 5 - $\sigma_x$ stress field in a plane parallel with OXZ, having Y = 10 mm;
- in figure 6 - $\tau_{xz}$ stress field in a plane parallel with OXZ, having Y = 10 mm;
- in figure 7 - $\sigma_x$ stress field in a plane parallel with OYZ, having X = 120 mm.

In figures 7, the variations of von Mises $\sigma_{eq}$ (fig. 7.a), $\sigma_x$ (fig. 7.b) and $\tau_{xz}$ (fig. 7.c) stresses along the thickness of the plate (OY direction), on the vertical line starting from point A of the discontinuity are plotted (fig. 5).

Example of a twisted 3D plate with a discontinuity in the resin layer.

Another example is further presented: a plate three dimensionally twisted, as it is shown in figure 8.

The model of the plate had 664 nodes and 2250 elements. The stress fields in the plate are presented in the next figures as follows:
- in figure 9, von Mises stress field in the projection on a plane parallel with OXZ (upper view);
- in figure 10, $\sigma_x$ stress field in the projection on a plane parallel with OXZ (upper view);
- in figure 11, von Mises stress field in orthogonal view;
- in figure 12, $\sigma_x$ stress field in a plane parallel with OYZ, having X = 150 mm;
- in figure 13, $\sigma_x$ stress field in a plane parallel with OXY, having Y = 50 mm;
- in figure 14, $\sigma_x$ stress field in a plane parallel with OXY, having Z = 120 mm.
Modellings and analysis of junctions for structures made from laminated composites

In the case of junctions the main idea is to find a technological and designing solution focused on the possibilities of composite layers interpenetration, in order to assure for this area the maximum strength for a given loading. The next step is linked to the optimization of geometrical and designing configurations for the respective junctions.

The authors propose a geometrical and designing methodology of the laminated composite structures junctions. In the paper are presented some modelling and analysis examples made through the Finite Element Method for some junctions belonging to process equipment.

Constructive examples

Some simple and frequently encountered junctions are presented in different designing variants. For each one, depending on the functional, technological and economical factors, the optimal variant is chosen.

In figure 15 four variants of a simple junction (variation of the external diameter from 2R₁ to 2R₂, with the internal diameter r) are represented for a tube manufactured from a composite material with six laminae (layers).

- the continuity of laminae is not ensured for variant a;
- partial continuity exists for variant b;
- all laminae have continuity in variant c;
- the junction of the composite material with a homogeneous one is shown in variant d.

Different variants of a junction between a tube and a flange are depicted in figure 16. The tube has an external diameter 2R₁, internal diameter 2r and the flange has a thickness h and external diameter 2R. The composite material is made of three laminae for the tube and seven for the flange. One can notice that, in variant a where there is no interpenetration between the layers of the composite belonging to the flange and the tube (the flange is outside the tube and is glued with an adhesive), the mechanical strength of the structure is small. Delamination may occur for the external layer of the tube.

For each of the above mentioned variants, homogeneous elements for reinforcement may be introduced. For example, for an aluminum - resin composite material, a steel reinforcement is shown in black in figures 16, b...f.
For a more complicated junction, like the one between the cylindrical shell of a tank (with diameters 2R₁ and 2R₂) and a lateral nozzle (with diameters 2r₁ and 2r₂), five variants are presented in figure 17. Similar observations as for the previous cases may be underlined. One can remark variant c in which a ring is used.

Modelling and analysis of junctions

In the case of junctions, modeling and analysis have as the main goal determination of the stresses in the layers and at the interface between layers. If one uses the finite element method, special elements for different types of composites are used. The most efficient and used method which could be successfully applied in this case is the Finite Element Method (FEM).

A specific modelling is necessary as for local problems. This means that one should use a refined mesh, in order to take into account the geometry and properties of each layer. In this way, information regarding the values of the interface stresses may be obtained, in order to assess the possible failure or delamination.

An example of finite element modeling and analysis for the structure in figure 16,c is further presented in order to illustrate the previous assessments. The following dimensions were considered: R₁ = 65 mm, r = 50 mm, R₂ = 120 mm, h = 30 mm, for an aluminum epoxy resin composite material. All layers have a thickness of 5 mm. A steel ring was inserted (drawn in black in fig.18) in order
to increase the strength and stiffness of the flange. The elastic constants are:

- \( E = 70000 \text{ N/mm}^2, G = 25000 \text{ N/mm}^2, \nu = 0.35 \) for aluminum
- \( E = 2440 \text{ N/mm}^2, G = 1200 \text{ N/mm}^2, \nu = 0.46 \) for resin
- \( E = 2.1 \times 10^5 \text{ N/mm}^2, G = 8.1 \times 10^4 \text{ N/mm}^2, \nu = 0.3 \) for steel.

An internal uniform pressure of 5 N/mm\(^2\) was applied to the tube. Constraints were applied on the frontal surface of the flange.

An axial-symmetric model was considered, using triangular elements. Each layer was separately modeled in order to introduce different elastic constants. The model has 243 nodes and 388 elements and is shown in figure 19.

Some results of practical interest are shown in figure 20. The values of von Mises equivalent stress \( \sigma_{\text{eq}} \) (fig.20, b), normal stress \( \sigma_{y} \) (fig.20, c) and shear stress \( \tau_{xy} \) (fig.20, d), were calculated in the nodes lying on the interface between two layers (drawn with a thick line in figure 20, a).

The stresses were defined with respect to the global system of axes (fig. 18). Using the values of these stresses, one may assess if the junction resists or not to the applied loads, based on specific failure criteria for composite materials [1].

Methodology for the optimization of the junctions in laminated composite structures

In order to successfully optimize a junction in a structure made of a laminated composite material, some aspects should be taken into account:

1. the local character of the analysis is the fundamental problem in the case of junctions. That is why a sub model or a sub structure of the junction is to be analyzed. This approach ensures the efficiency of the process;

2. a parametric mode is used, that is all dimensions will be denoted with letters. Some parameters will have constant values, other will be declared as design variables;

3. the constant values of the parameters must be established with discrimination in order to avoid geometric configuration with superposition, voids, distortions, etc;

4. the definition of the geometry (points, lines, surfaces, volumes) must be done as to emphasize the different layers, because these layers are made from different materials. The results obtained following a finite element analysis must give information regarding the interface stresses and strains;

5. the mesh must be obtained by generating the nodes and elements along each layer, as distinctive groups of elements with different physical, mechanical and elastic properties.

All other aspects of modeling, analysis and optimization are the usual ones for a finite element analysis.

Example of a junction optimization

A variant of a junction between a tube and a cylindrical flange (fig. 16, c and fig. 18) was chosen in order to illustrate the above mentioned optimization methodology. This junction is presented in figure 21. Three design variables were considered: H1, H2 and R1.

The composite material is aluminum - epoxy resin, having the same elastic constants as in the previous example (fig.18). The thickness of the layers were denoted as H1 and H2, as one can see in figure 21. A constant internal pressure \( p = 0.1 \text{ N/mm}^2 \) was applied on the tube.
An axial symmetric model was considered, using triangular elements. Each layer was separately modeled in order to introduce different mechanical characteristics. The model has 1202 nodes and 2140 elements (fig. 22).

The optimization function was the weight of the junction and the restriction was an imposed value for the equivalent von Mises nodal stress. Also, constant values were chosen for some parameters as: R = 50 mm, L = 4R, RF = 10R.

The support has been considered to be on the frontal surface of the flange.

The following values for the design variables were obtained: H1 = 4.5 mm, H2 = 26.6 mm and R1 = 8.4 mm. The maximum value of the equivalent stress was 47.4 N/mm² and the weight of the junction was obtained 156 N.

Some results are graphically presented in figure 23.

In this figure, results were plotted for a part of a longitudinal line on the internal surface of the tube (starting from the middle to the flange surface).

The variations of the equivalent von Mises stress σₑ, normal stress σₓ and σᵧ, and shear stress τₓᵧ are plotted, with respect to the global system of axes from figure 21.

Conclusions
A local problem, i.e. the existence of a discontinuity or junction presents an evident interest for mechanical engineering, due to the fact that the local stress state produced in the structure has a complex configuration, difficult to anticipate or to evaluate.

The modelling and the analysis of local problems could be done exclusively by the finite elements.

Since the finite element model should be adequate for the study of a local problem it must be elaborated by respecting a certain methodology; in this case the meshing must be done on each layer using 3D elements.

For laminated composite materials the information obtained by modelling and corresponding analysis could determine the cases where failure of composites could occur: layers fracture, delaminations etc.

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