Modelling the Thermal Interaction Between Soil and Different Geometries of Polyethylene Heat Exchangers

MIHAI ALBULESCU1, SORIN NEACSU1, CRISTIAN EPARU1*, MARCELA PATARLAGEANU1, FLORINEL DINU1, ADRIANA CONȚ2
1 Petroleum-Gas University of Ploiești, 39 București Bd., 100680, Ploiești, Romania
2 SC Ciocârlia SA

The extraction of surface geothermal energy using heat pumps represents a recent topic in specialty literature. This paper presents a numerical model for modelling the thermal field around the heat exchangers mounted in the soil. The purpose of modelling is to reveal the dynamics of heat exchange between the soil and the polyethylene pipes and highlighting the variation in temperature field around the heat exchanger.

Keywords: Geoexchange, heat pump, polyethylene, thermal field

An important renewable resource that can be used in heating dwellings and in the preparation of running hot water is the surface geothermal energy, which has two features: high heat capacity and a low average temperature between 10 and 16°C. Heat pumps are required in order to use this type of resource. These heat pumps extract the thermal energy from the soil through a circuit made up of polyethylene pipes installed in the soil in different geometries (fig. 1), a system which is called Geoexchange.

A mix of water and glycol circulate between the heat pump evaporator and the soil through polyethylene pipes. Within the evaporator, the water-glycol mix provides the necessary heat to vaporize the thermodynamic agent of the heat pump, usually Freon, hence the mix cools with 3-4°C. The cold mix between water and glycol is circulated through pipes installed in the soil in order to absorb heat and carry this heat to the evaporator. It is known that the heat of a heat pump is provided at a rate of 75-80% from the heat extracted from the cold source, in this case the soil [4, 6].

This paper presents an efficient numerical model for modelling the thermal field around the heat exchangers mounted in the soil. The configuration of the temperature field in time and space is determined by the ability of the heat exchanger to extract the thermal energy from the soil. The heat taken via the polyethylene pipes is directly proportional to the temperature field gradient in the pipe wall [3, 5].

The numerical model was developed for the geometries of heat exchangers mounted in the experimental polygon of Petroleum-Gas University of Ploiesti and was validated by means of measurements of the temperature field in the vicinity of the pipelines [1, 2].

The numerical model is based on two systems of equations depending on the characteristics of the thermal phenomena [1, 7]:
- non-isothermal flow equations of liquids through pipelines;
- heat diffusion equations around the pipeline.

The model was designed to allow simulation of various geometries of pipes placed in soil.

Modelling non-isothermal water flow through polyethylene pipes

The mathematical modelling of non-isothermal flow is based on the energy balance of the flow, in the time unit, which means that a balance of powers is made.

Let us take into consideration a pipe with diameter \( d \) divided into elements of constant volume, with equal length \( \Delta x \) (fig. 2). Each element has a constant temperature throughout the volume, denoted by \( T_1, T_2, \ldots, T_i, \ldots \), considered in the mass center of volume elements.
The following thermal powers are being defined:

- \( Q_s \) - the thermal power received by the volume element from the pipe wall
  \[
  Q_s = \pi d \Delta x (T_p - T_i) \tag{1}
  \]
- \( Q_1 \) - the thermal power that is carried into the volume element by the fluid.
  If the flow occurs from left to right, then the liquid comes with the temperature \( T_{i-1} \).
  \[
  Q_1 = w \rho c T_{i-1} \tag{2}
  \]
- \( Q_2 \) - the thermal power leaving the volume element due to fluid flow
  \[
  Q_2 = w \rho c T_i \tag{3}
  \]
- \( Q_{ac} \) - the heat accumulated in the volume element
  \[
  Q_{ac} = \frac{\pi d^2 \Delta x \rho c (T_{i+1}^{n+1} - T_i^n)}{\Delta t} \tag{4}
  \]

If we consider time to be divided into elementary time intervals, \( \Delta t \) size, the elementary volume temperature at the current moment of time noted with \( T_i^n \) will become \( T_i^{n+1} \) at the following moment of time.

The power balance for the elementary volume \( i \) of the pipe is

\[
\frac{\pi d^2 \Delta x}{4} \rho c \left( \frac{T_{i+1}^{n+1} - T_i^n}{\Delta t} \right) = \pi d \Delta x (T_p^n - T_i^n) + w \rho c T_{i-1}^{n+1} - w \rho c T_i^n \tag{5}
\]

The temperatures of the right member of equation (5) were considered at the current time \( n \). In this situation, if that formula applies to all elementary volumes, a linear system of equations follows. From each equation the elementary volume temperature can be explicit to the next time, \( n+1 \). The system of linear equations generated by the relationship (5) can be solved through other adequate methods.

**Modeling the equation of diffusion occurring in the soil around the polyethylene pipe**

The method of finite volumes has been preferred to model heat diffusion, a method through which the soil is divided into elementary volumes with a constant temperature. The geometric point where the temperature is considered is the mass center of the volume.

In order to simplify the calculus, a parallelepiped volume with measuring sides \( \Delta x, \Delta y \) and \( \Delta z \) has been considered (fig. 3).

The balance of heat flows on the elementary parallelepiped faces is being performed, and considering that we do not have heat sources, the temperature values in the network point are written as a linear combination between moments of time \( n \) and \( n+1 \).

\[
T_{ij,k}^{n+1} = \beta T_{ij,k}^n + \frac{\Delta x}{4} \left( T_{i-1,j,k}^{n+1} + T_{i+1,j,k}^{n+1} + T_{i,j-1,k}^{n+1} + T_{i,j+1,k}^{n+1} \right) + \alpha \frac{\Delta t}{\Delta x^2} \left( T_{i+1,j,k}^n + T_{i,j+1,k}^n - 2T_{ij,k}^n \right)
\]

where \( 0 \leq \gamma \leq 1 \).

By applying expression (6) in all network points, results a linear system of equations. The number of equations equals the number of unknowns. The number of equations reported to unknown number equals the number of nodes in the network.

By customizing the value of \( \gamma \) we can obtain [1]:

- for \( \gamma = 1 \), the fundamental default method follows;
- for \( \gamma = 0.5 \), the Kranck – Nicolson method follows;
- for \( \gamma = 0 \), the fundamental explicit method follows.

For any value of \( \gamma \) between \( (0, 1) \), relationship (6) generates a system of equations that can be solved directly by Gauss method or iteratively by a Gauss-Siedel method. When the number of equations is significant, iterative methods are preferred in order to shorten the calculation.

The following notations have been used:

- \( \rho \) - density of the soil;
- \( c \) - specific heat of the soil.

\[
\alpha = \frac{\lambda}{\rho c} \tag{7}
\]

\[
\beta = 1 - 2\alpha \left( \frac{\Delta t}{\Delta x^2} + \frac{\Delta t}{\Delta y^2} + \frac{\Delta t}{\Delta z^2} \right) \tag{8}
\]

\( \alpha \) representing the coefficient of thermal diffusivity.
Coupling the flow equation with the heat diffusion equation in order to achieve the numerical model

In order to simulate the process of extracting heat from soil, the water flow equation in non-isothermal state through pipes installed in the soil must be coupled with the heat diffusion equation via a limit condition and simultaneously solved.

Coupling the two equations allows the simulation of cold water heating process which comes from the heat pump evaporator, as it flows through pipes installed in the soil. At the same time, the soil around the pipe cools [2, 5].

The limit condition at the pipe wall has been considered the best possible coupling of the two systems of equations. In order to achieve this, the pipe route has been defined in relation with the mesh network, so as to always pass through nodes of the elementary volume in which the soil has been divided.

A cross section on the axis Oz through the elementary parallelepiped and the pipe axis is shown in figure 4.

![Diagram](image)

Fig. 4.

The temperature in the center of the elementary parallelepiped \( \Delta x \Delta y \Delta z \), which the pipeline crosses, is \( T_{i,j,k} \). Let us consider that this temperature is equal to the pipe wall temperature \( T_p \), temperature that can be determined from the limit condition.

In order to define the limit conditions, the power balance equation will be applied for the pipe wall. According to this equation, the thermal power gained by the pipe wall is the difference between the thermal power received by conduction from adjacent volumes and the thermal power taken by water flowing through the pipe. Using the notations of figure 4 the following relationship can be written:

\[
\dot{Q}_a = \lambda \Delta x \Delta z \left( \frac{T_{i+1,j,k} - T_{i,j,k}}{\Delta y - \frac{d_x}{2}} + \lambda \Delta y \Delta z \frac{T_{i+1,j,k} - T_{i,j,k}}{\Delta x - \frac{d_y}{2}} + \lambda \Delta x \Delta z \frac{T_{i,j+1,k} - T_{i,j,k}}{\Delta y - \frac{d_y}{2}} + \lambda \Delta y \Delta z \frac{T_{i,j+1,k} - T_{i,j,k}}{\Delta x - \frac{d_x}{2}} \right )
\]

where \( \dot{Q}_a \) is the thermal power from adjacent volumes. The heat flow along the axis Oz has been neglected.

The thermal power taken by water is:

\[
\dot{Q}_p = \pi \frac{d_y^2 - d_x^2}{4} \Delta z \, \rho_p \, c_p \, \frac{T_p^{n+1} - T_p^n}{\Delta t}
\]

where:
- \( T_p \) - the wall temperature
- \( T_p^0 \) - water temperature
- \( a \) - coefficient of thermal convection
- \( \rho_p \) - polyethylene density
- \( c_p \) - polyethylene specific heat

Using the above assumption (the temperature in the center of the elementary parallelepiped, which is crossed by the pipe, equals the pipe wall temperature) and applying the power balance equation, the following results:

\[
\dot{Q}_a = \pi \frac{d_y^2 - d_x^2}{4} \Delta z \, \rho_p \, c_p \, \frac{T_p^{n+1} - T_p^n}{\Delta t}
\]

Equation (12) is valid only for the nodes through which the pipe passes. The limit condition at the pipe wall can be determined using equation (12).

Results and discussions

Some graphical results made in DELPHI development environment are being presented in the following section. The figures represent the thermal field around the heat exchanger at different times. From a space point of view, a graphical representation of temperature from a horizontal level containing the heat exchanger has been chosen.

Figures 5a, 6a, 7a and 8a represent top views, while figures 5b, 6b, 7b and 8b represent the side views of temperature from a horizontal level containing the

Fig. 5a - time: 500 s
polyethylene pipe.

The soil temperature is 16°C and in figures 7b and 8b one can easily observe the changes in water temperature in pipes from 12°C at the entrance to almost 16°C.

As time goes by, the soil around the pipe cools, which can be easily observed through the enlarged area of thermal influence around the pipe.

Conclusions

In these study cases, two types of polyethylene heat exchangers geometry mounted in the soil have been chosen. The numerical model based on the equations described above in DELPHI development environment, due to the implementation characteristics, is able to model any geometry of heat exchanger mounted in the soil, as it can be seen in paper [1].

The thermal field variation in time and space around polyethylene heat exchangers mounted in the soil can be obtained by using the numerical model. This makes possible the determination of the value of the derivative thermal field at pipe wall at any point of the heat exchanger, a value which allows calculating the exact amount of heat flow over the pipe length, at different times.

The numerical analyses performed with this model have shown that the thermal power extracted from the soil decreases with the exploitation time of the heat exchanger, due to changes in the temperature gradient at the pipe wall, which can be easily seen in the graphics presented.

References

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Manuscript received: 1.02.2010